



An electro-thermo-mechanical beam model for structural health monitoring of new ice protection laminate

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Motivations, gaps and objectives

Motivations

- Trending application of electric IPS system in aerospace industries
- Prospects of electric IPS for real-time sensing functions
- Prospects of analytical modelling framework for SHM

Research gaps

- Challenging optimisation of power consumption in electric IPS
- Complexities in integration of sensors with IPS
- Computational complexities in structural analysis of IPS with embedded sensors

Objectives

- Develop novel analytical framework for structural analyses of IPS with embedded sensors
- Integrate the effect of bulk and delamination damages in IPS model
- Structural analyses and 3D FE validation of IPS system
- SHM analysis of IPS system



Kinematical formulations

3 parameter displacement relations

$$u(x, y, z, t) = u_0(x, y, t) + z\varphi_0(x, y, t),$$

$$w(x, y, z, t) = w_0(x, y, t),$$

5 parameter displacement relations

$$u(x, y, z, t) = u_0 + f_{z1}(z)\bar{w}_0 + f_{z2}(z)\varphi_0 + f_{z3}(z)\psi,$$

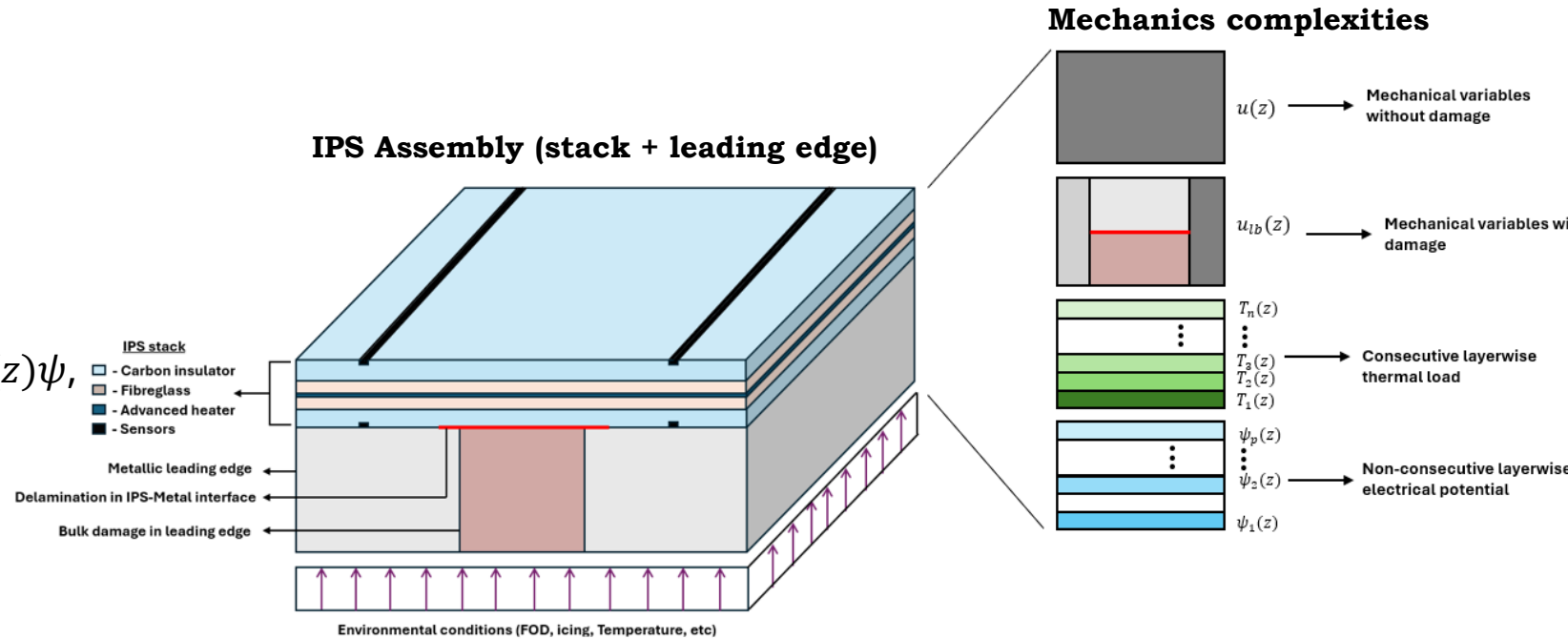
$$w(x, y, z, t) = w_0(x, y, t),$$

Thickness functions

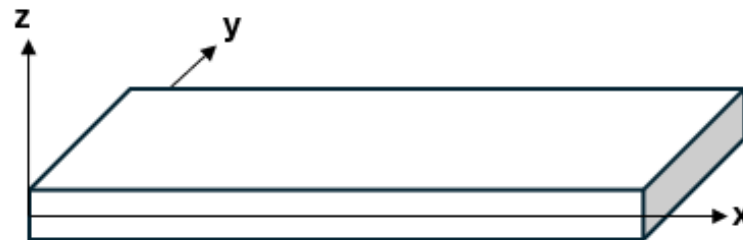
$$f_{z1} = -\frac{4}{3h^2}z^3,$$

$$f_{z2} = \left(z - \frac{4}{3h^2}z^3\right),$$

$$f_{z3} = \frac{(h/\pi) \sinh\left(\frac{\pi z^k}{h}\right) - z^k}{\cosh\left(\frac{\pi}{2}\right) - 1}.$$



Beam reference



Kinematical assumptions

- Thickness is smaller than in-plane dimensions.
- Rigid cross-section (plane strain).
- Transverse loading application.

Constitutive relations

$$\begin{Bmatrix} \sigma_\theta \\ \sigma_\vartheta \\ \sigma_z \\ \tau_{\vartheta z} \\ \tau_{\theta z} \\ \tau_{\theta\vartheta} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11}^k & \bar{Q}_{12}^k & \bar{Q}_{13}^k & 0 & 0 & \bar{Q}_{16}^k \\ \bar{Q}_{12}^k & \bar{Q}_{22}^k & \bar{Q}_{23}^k & 0 & 0 & \bar{Q}_{26}^k \\ \bar{Q}_{13}^k & \bar{Q}_{23}^k & \bar{Q}_{33}^k & 0 & 0 & \bar{Q}_{36}^k \\ 0 & 0 & 0 & \bar{Q}_{44}^k & \bar{Q}_{45}^k & 0 \\ 0 & 0 & 0 & \bar{Q}_{45}^k & \bar{Q}_{55}^k & 0 \\ \bar{Q}_{16}^k & \bar{Q}_{26}^k & \bar{Q}_{36}^k & 0 & 0 & \bar{Q}_{66}^k \end{bmatrix} \begin{pmatrix} \begin{Bmatrix} \varepsilon_\theta \\ \varepsilon_\vartheta \\ \varepsilon_z \\ \gamma_{\vartheta z} \\ \gamma_{\theta z} \\ \gamma_{\theta\vartheta} \end{Bmatrix}^k - \begin{Bmatrix} \alpha_\theta \\ \alpha_\vartheta \\ 0 \\ 0 \\ 0 \\ \alpha_{\theta\vartheta} \end{Bmatrix}^k \Delta T_z \end{pmatrix} - \begin{Bmatrix} \bar{e}_{31} \\ \bar{e}_{32} \\ \bar{e}_{33} \\ 0 \\ 0 \\ \bar{e}_{36} \end{Bmatrix}^k E_z$$

Elastic constants

Piezoelectric constants

Dielectric constants

Expansion coefficients

$$D_z = \begin{bmatrix} \bar{e}_{31} & \bar{e}_{32} & \bar{e}_{33} & 0 & 0 & \bar{e}_{36} \end{bmatrix} \begin{Bmatrix} \varepsilon_\theta \\ \varepsilon_\vartheta \\ \varepsilon_z \\ \gamma_{\theta z} \\ \gamma_{\vartheta z} \\ \gamma_{\theta\vartheta} \end{Bmatrix}^k + \eta_{33}^k E_z$$

Constitutive assumptions

- Negligible transverse normal stress.
- Negligible in-plane shear stress
- Damage dependent material coefficients

Refined constitutive relations

1D constitutive equation with bulk damage effect

$$\begin{bmatrix} \sigma_{xx}^k \\ \tau_{xz}^k \\ D_z^k \end{bmatrix} = \begin{bmatrix} \hat{Q}_{11}^k & 0 & \hat{\Theta}_{11}^k & -\hat{Q}_{\alpha 1}^k \\ 0 & k_s \hat{Q}_{55}^k & 0 & 0 \\ -\hat{\Theta}_{11}^k & 0 & \hat{\eta}_{33}^k & \hat{Q}_{\alpha 2}^k \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^k \\ \gamma_{xz}^k \\ E_z^k \\ \Delta T_z \end{bmatrix}$$

$\hat{Q}_{11}^k(\Gamma)$, $\hat{Q}_{55}^k(\Gamma)$, $\Gamma \in [0, 1]$ - damage dependent stiffnesses

$\Gamma_i = 1 - \frac{A_i^*}{A_i}$, $i = 1, 2, 3$ - anisotropic damage parameters

A_i^* and A_i represent damaged and undamaged areas in the orthotropic material

Effective material coefficients

$\hat{Q}_{11}^k$ - longitudinal stiffness coefficient

$\hat{\Theta}_{11}^k$ - piezoelectric coupling

$\hat{Q}_{55}^k$ - shear coefficient

$\hat{\eta}_{33}^k$ - piezoelectric coefficient

$\hat{Q}_{\alpha 1}^k$ - mechanical thermal expansion coefficient

$\hat{Q}_{\alpha 2}^k$ - electrical thermal expansion coefficient

Semi-layerwise equilibrium equations

Governing equations

$$\begin{bmatrix} m_{uu} & m_{uw} & m_{u\varphi} \\ m_{wu} & m_{ww} & m_{w\varphi} \\ m_{\varphi u} & m_{\varphi w} & m_{\varphi\varphi} \end{bmatrix}_{bl} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{w}_0 \\ \ddot{\varphi}_0 \end{Bmatrix}_{bl} + \begin{bmatrix} k_{uu} & k_{uw} & k_{u\varphi} \\ k_{wu} & k_{ww} & k_{w\varphi} \\ k_{\varphi u} & k_{\varphi w} & k_{\varphi\varphi} \end{bmatrix}_{bl} \begin{Bmatrix} u_0 \\ w_0 \\ \varphi_0 \end{Bmatrix}_{bl} + \begin{bmatrix} \{h_{u\psi}\} \\ \{h_{w\psi}\} \\ \{h_{\varphi\psi}\} \end{bmatrix}_{bl} \{\bar{\Psi}\}_{bl} + \begin{bmatrix} \{q_{uT}\} \\ \{q_{wT}\} \\ \{q_{\varphi T}\} \end{bmatrix}_{bl} \{\Delta T\}_{bl} - \begin{bmatrix} \sum_{i=1}^{N_i} \bar{\tau}_{xz}^{(i)} \\ \sum_{i=1}^{N_i} \bar{\sigma}_{zz}^{(i)} \\ z^i \sum_{i=1}^{N_i} \bar{\tau}_{xz}^{(i)} \end{bmatrix} = \begin{bmatrix} 0 \\ \{F_z\} \\ 0 \end{bmatrix}_b$$

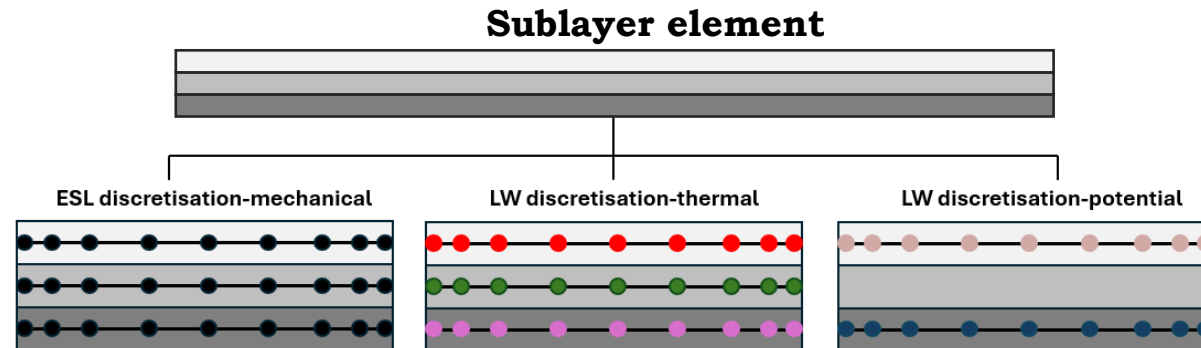
$$\begin{bmatrix} g_{\psi u} & g_{\psi w} & g_{\psi\varphi} \end{bmatrix}_{bl} \begin{Bmatrix} u_0 \\ w_0 \\ \varphi_0 \end{Bmatrix}_{bl} + \begin{bmatrix} h_{\psi\psi} \end{bmatrix}_{bl} \{\bar{\Psi}\}_{bl} + \begin{bmatrix} q_{TT} \end{bmatrix}_{bl} \{\Delta T\}_{bl} = 0$$

$$\left[B_{11} \frac{\partial^2 u_0}{\partial x^2} + \bar{B}_{11} \frac{\partial^2 \bar{w}_0}{\partial x^2} + \bar{D}_{11} \frac{\partial^2 \varphi_0}{\partial x^2} + \bar{E}_{11} \frac{\partial^2 \psi}{\partial x^2} - B_{55} \frac{\partial w_0}{\partial x} - \bar{B}_{55} \bar{w}_0 - \bar{D}_{55} \varphi_0 - \bar{E}_{55} \psi - I_1 \ddot{u}_0 - I_{11} \ddot{\bar{w}}_0 - I_{12} \ddot{\varphi}_0 - I_{13} \ddot{\psi} - \left[B_{11}^{me1} \right] \frac{\partial \{\bar{\Psi}\}}{\partial x} - \left[H_{\alpha 1} \right] \frac{\partial \{\Delta T\}}{\partial x} \right]_{bl} - \sum_{i=1}^{N_i} f_{z1}^{(i)} \bar{\tau}_{xz}^{(i)} = 0$$

Numerical Implementation

Differential quadrature method

$$u(x) = \sum_{i=1}^N l_i(x) \tilde{u}_i$$



Sub-layer ESL variables

Sub-layer penalty variables

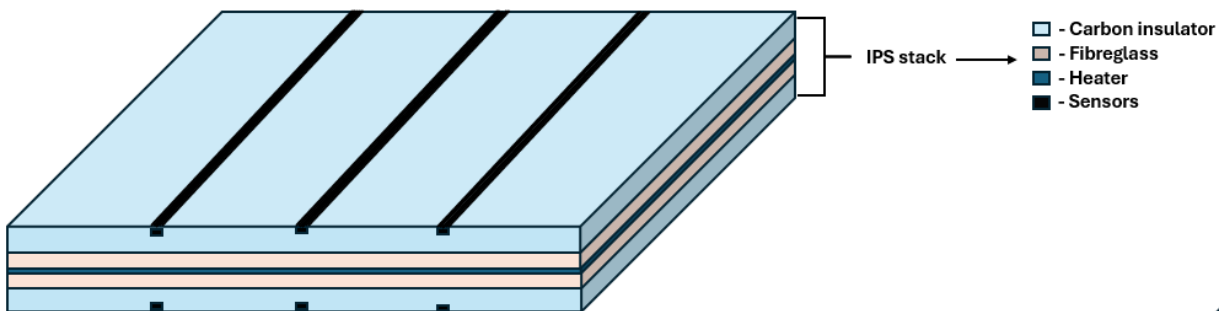
Sub-layer layerwise variables

Multidomain and layerwise summation

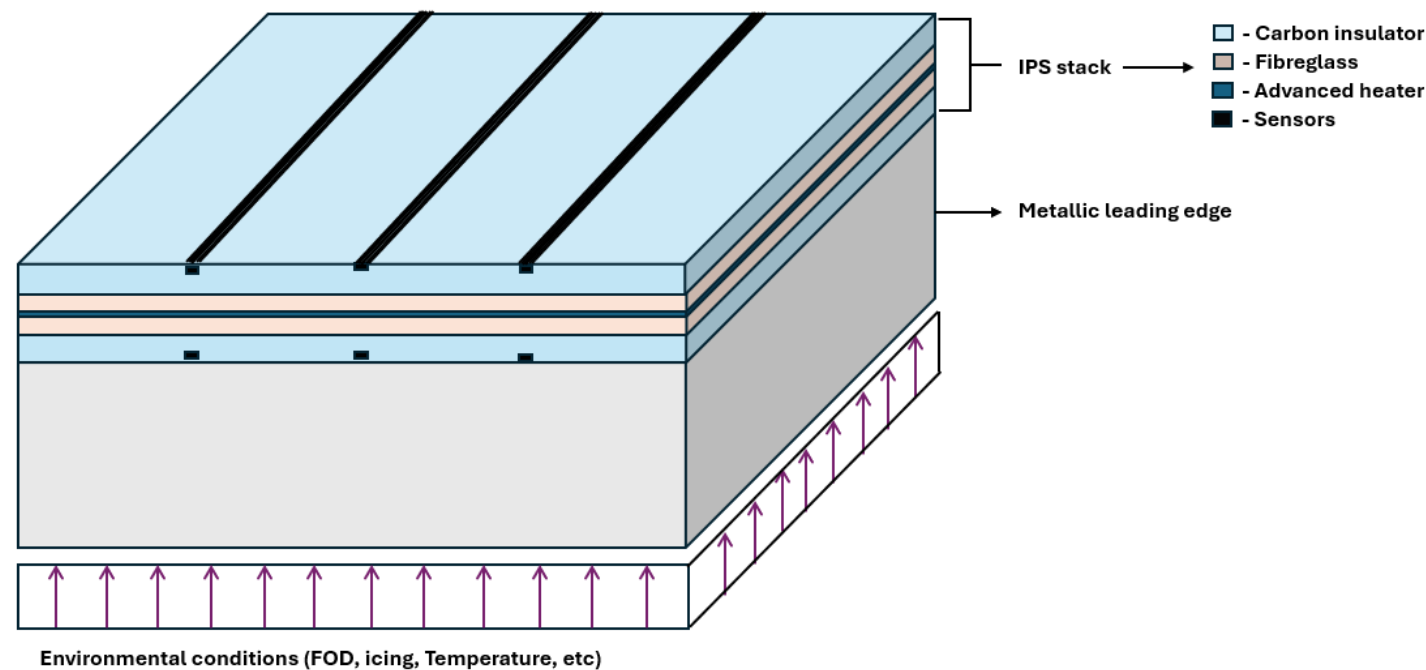
Numerical examples

Geometric configurations

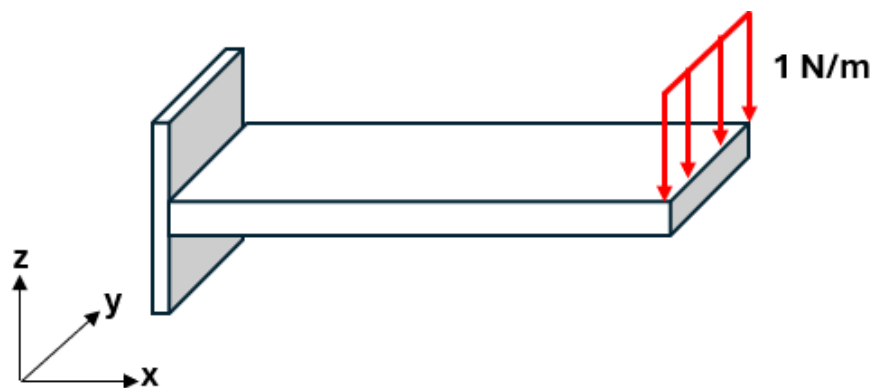
IPS stack



IPS Assembly (stack + metallic leading edge)



Test setup

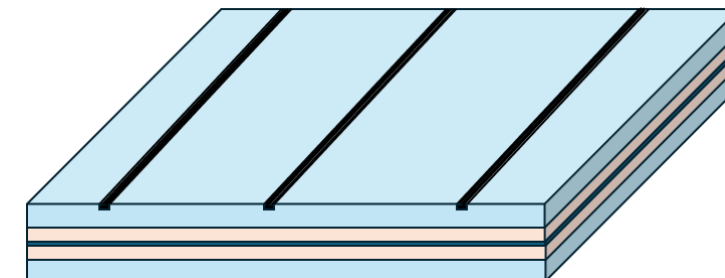


Thermal analysis

Layerwise temperature for one CNT-zone IPS

Layer	Start	Peak	Steady state
CI-top	2.4°C	-2.8°C	-7°C
FG-top	11.3°C	-1.8°C	-7°C
CNT	20.2°C	-0.8°C	-7°C
FG-bottom	18.9°C	-0.6°C	-7°C
CI-bottom	17.6°C	-0.4°C	-7°C

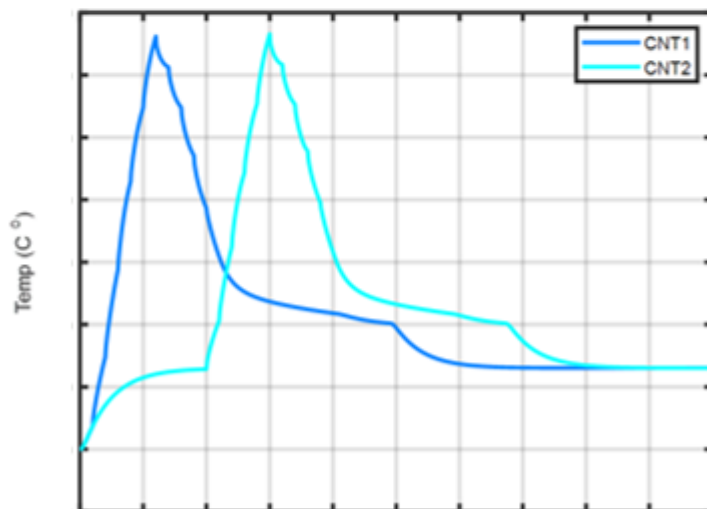
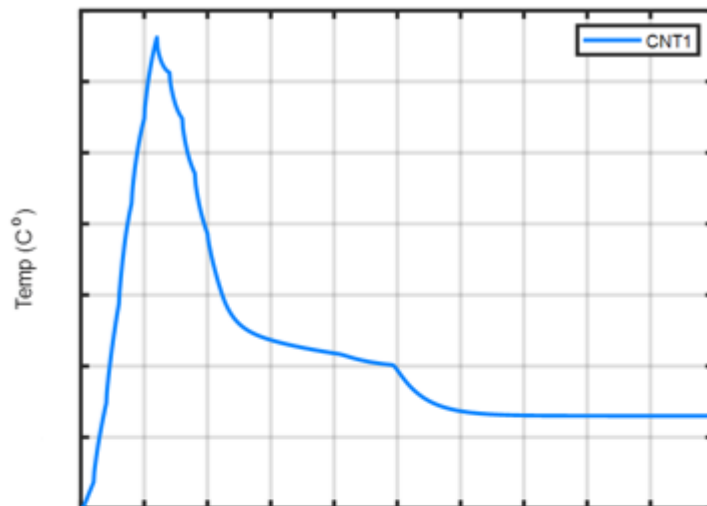
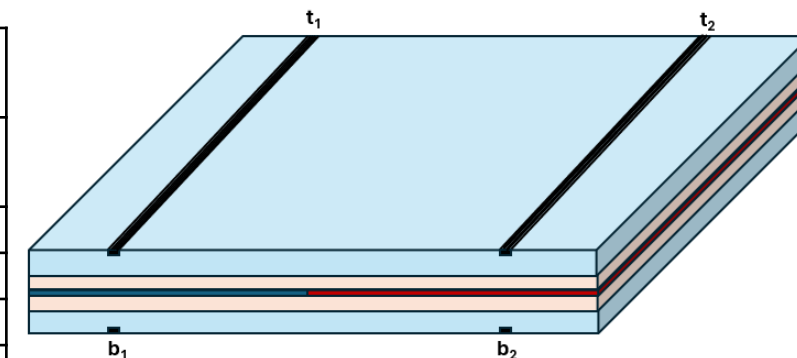
One CNT-zone IPS



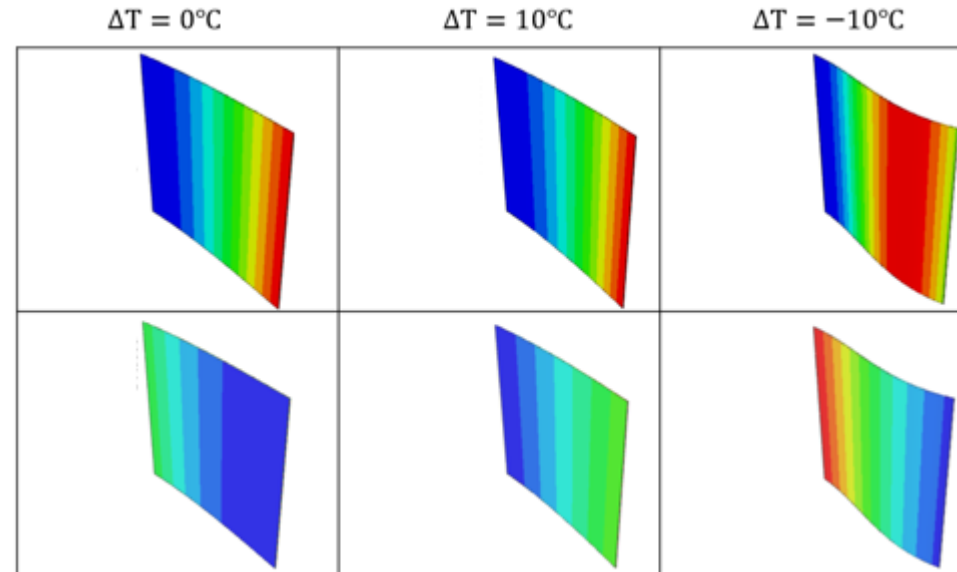
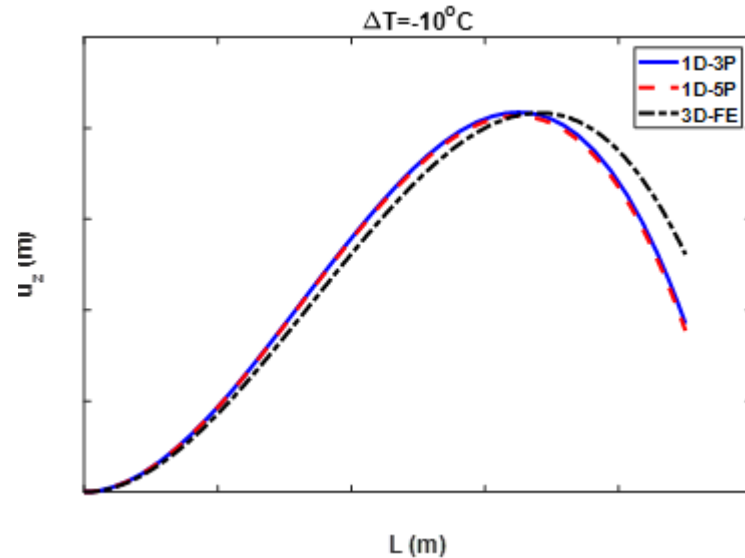
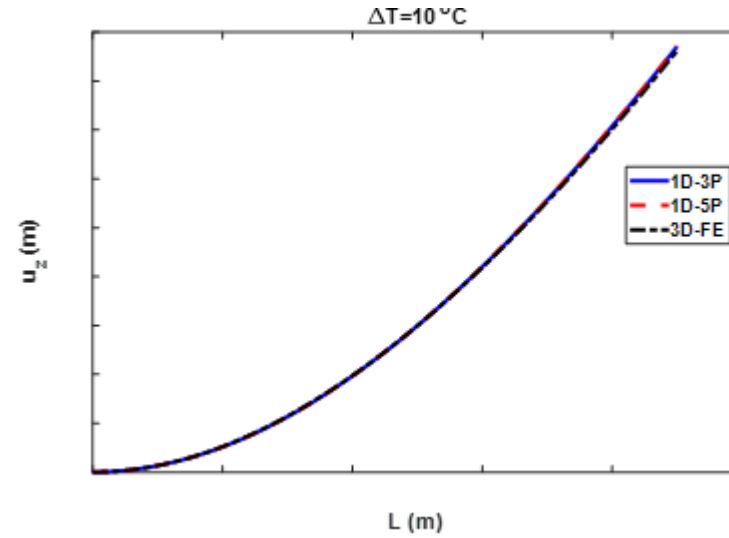
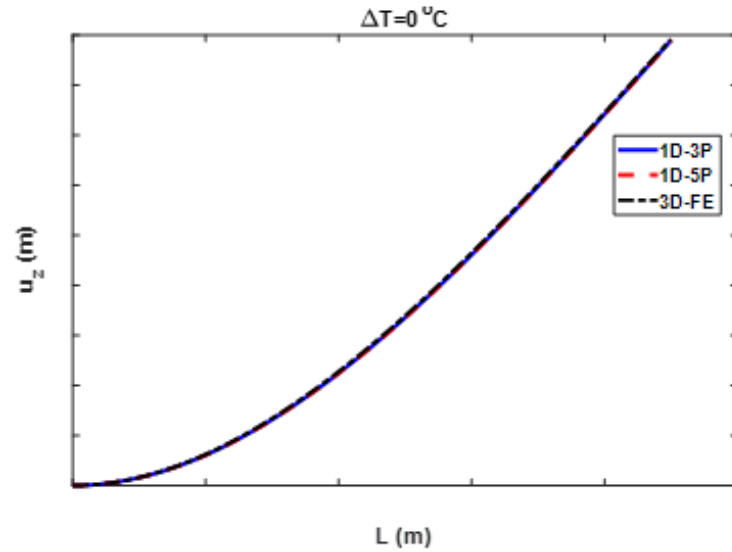
Layerwise temperature for two CNT-zone IPS

Layer	Start		Peak	
	CNT-zone-1	CNT-zone-2	CNT-zone-1	CNT-zone-2
CI-top	2.4°C	-8.7°C	-2.8°C	2.8°C
FG-top	11.3°C	-9.2°C	-1.8°C	3°C
CNT	20.2°C	-9.7°C	-0.8°C	3.2°C
FG-bottom	18.9°C	-9.9°C	-0.6°C	3.3°C
CI-bottom	17.6°C	-10°C	-0.4°C	3.3°C

Two CNT-zone IPS

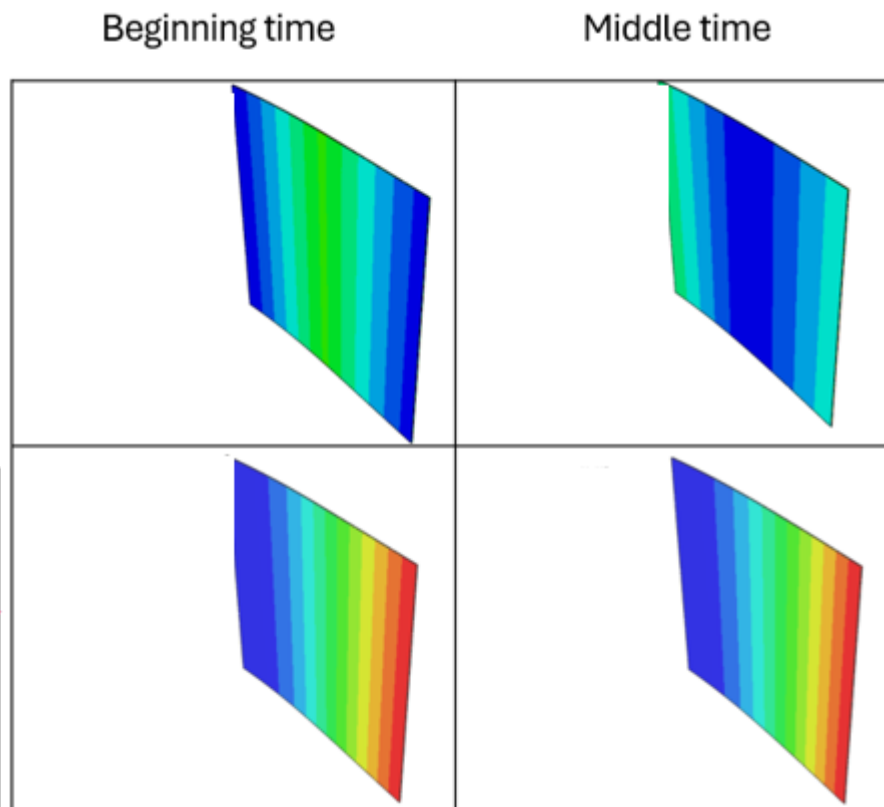
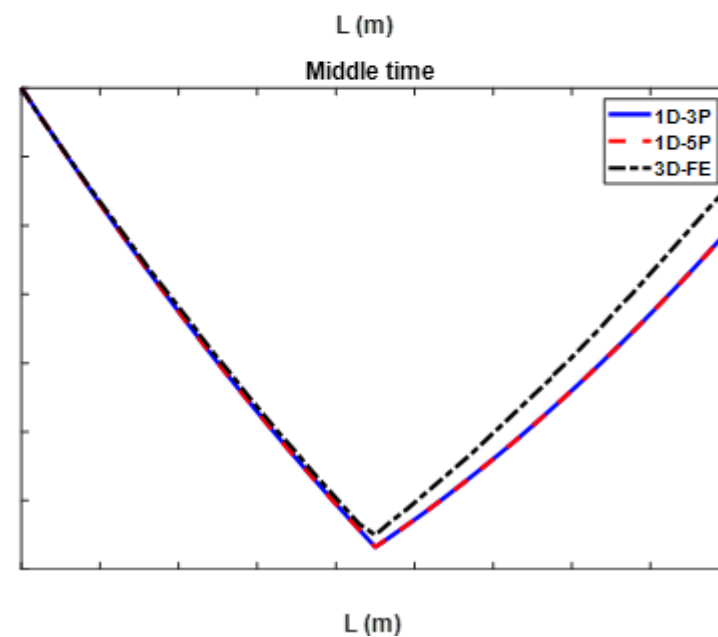
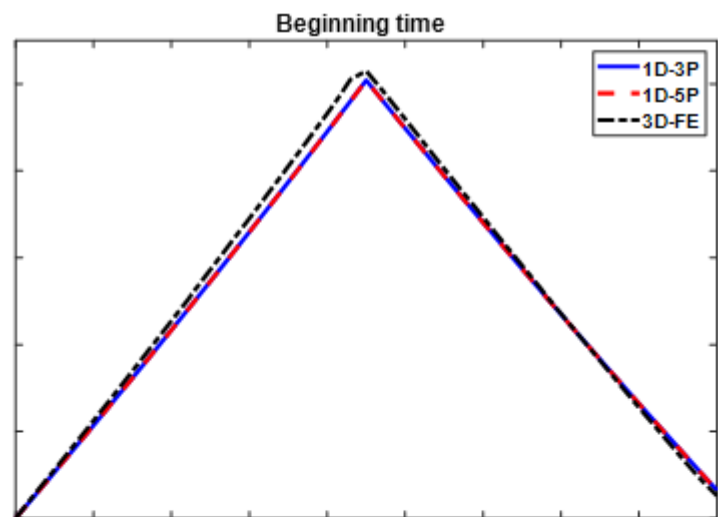


Static analysis – one CNT-zone IPS



Electric potential (V)			
	sensor1	sensor2	sensor3
$\Delta T = 0^\circ\text{C}$			
1D_3P	0.39	0.26	0.13
1D_5P	0.39	0.26	0.13
3D_FE	0.39	0.26	0.13
$\Delta T = 10^\circ\text{C}$			
1D_3P	-1.83	-1.96	-2.09
1D_5P	-1.84	-1.97	-2.11
3D_FE	-1.81	-2.15	-2.14
$\Delta T = 20^\circ\text{C}$			
1D_3P	-4.05	-4.19	-4.32
1D_5P	-4.08	-4.21	-4.34
3D_FE	-4.07	-4.58	-4.42
$\Delta T = -10^\circ\text{C}$			
1D_3P	2.62	2.49	2.35
1D_5P	2.63	2.5	2.37
3D_FE	3.07	2.56	2.45
$\Delta T = -20^\circ\text{C}$			
1D_3P	4.84	4.71	4.58
1D_5P	4.91	4.78	4.64
3D_FE	5.65	4.91	4.77

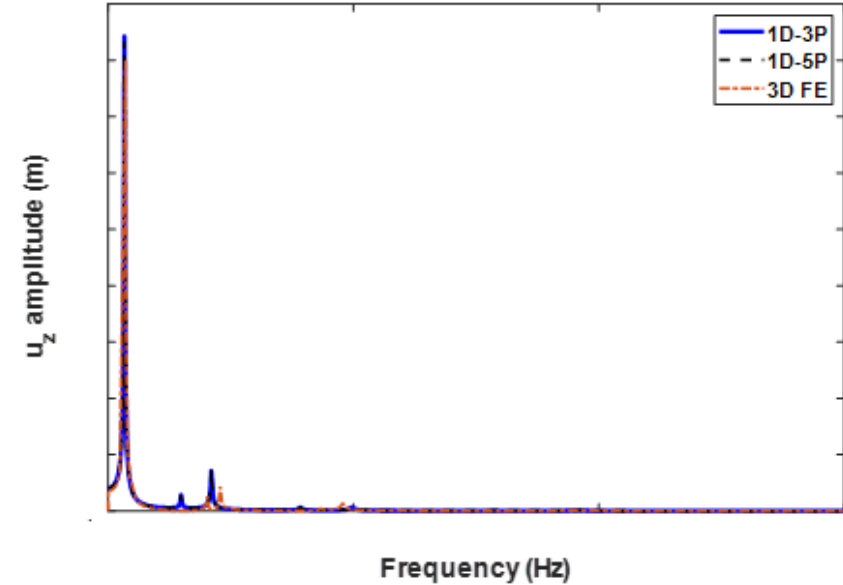
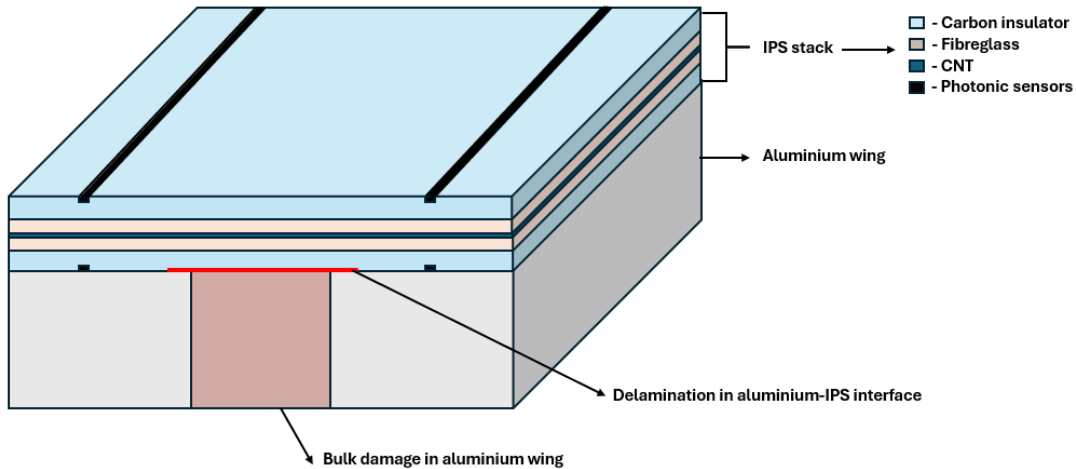
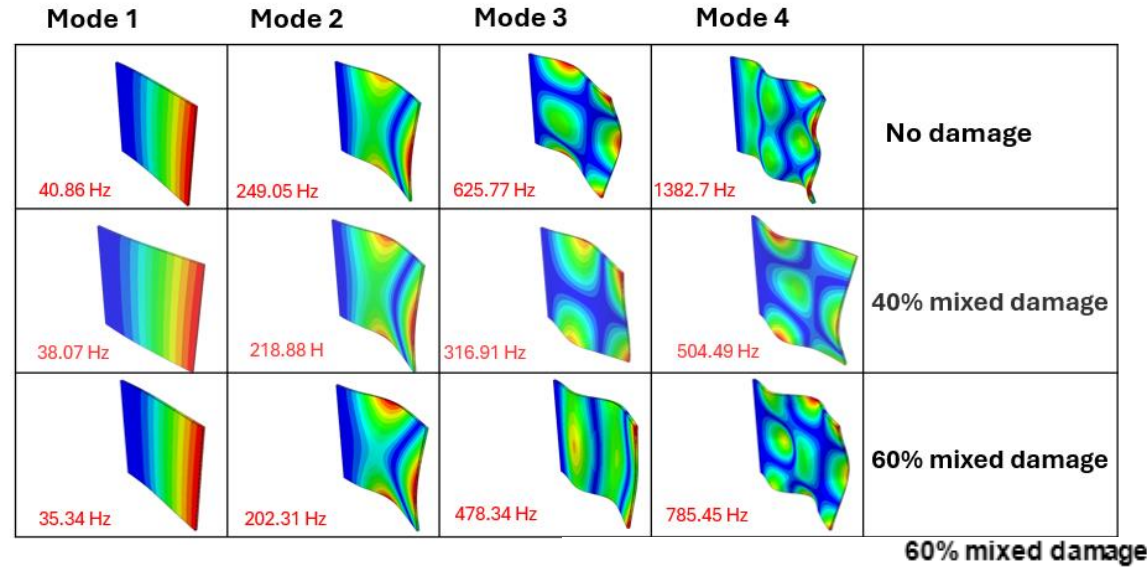
Static analysis – two CNT-zone IPS



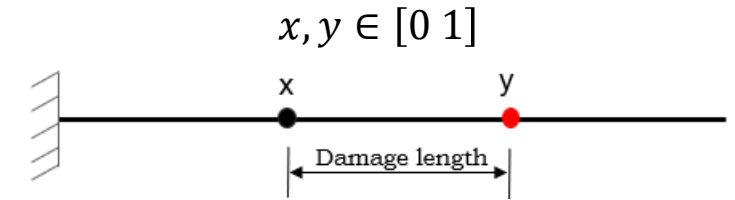
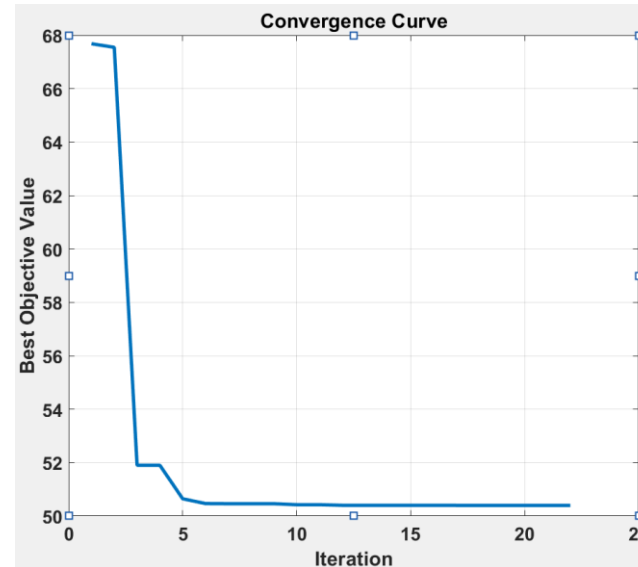
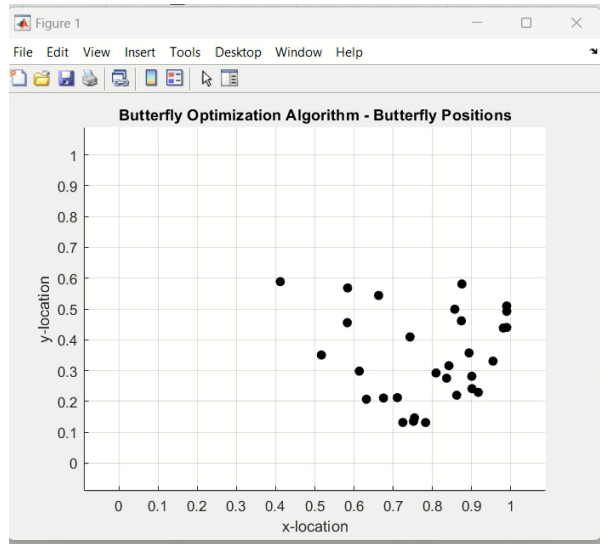
Electric potential (V)				
	sensor b ₁	sensor t ₁	sensor b ₂	sensor t ₂
	Start			
1D_3P	-2	2.22	-4.17	2.07
1D_5P	-1.96	2.22	-4.21	2.09
3D_FE	-3.25	2.63	-5.51	2.45
	Peak			
1D_3P	0.81	-0.56	-0.16	-0.86
1D_5P	0.82	-0.55	-0.18	-0.86
3D_FE	0.88	-0.62	-0.29	-1.02

Frequency analysis of IPS Assembly

	No damage			
	ω_1	ω_2	ω_3	ω_4
1D_3P	38.52	240.87	672.02	1310.12
1D_5P	38.52	240.9	672.2	1310.86
3D_FE	40.86	249.08	625.84	1382.7
	40% mixed damage			
	ω_1	ω_2	ω_3	ω_4
1D_3P	35.89	207.31	358.53	501.93
1D_5P	35.89	207.33	357.09	501.87
3D_FE	38.07	218.88	316.91	504.49
	60% mixed damage			
	ω_1	ω_2	ω_3	ω_4
1D_3P	33.33	210.54	497.66	787.84
1D_5P	33.33	210.41	497.11	786.05
3D_FE	35.34	228.45	478.34	785.45



SHM analysis

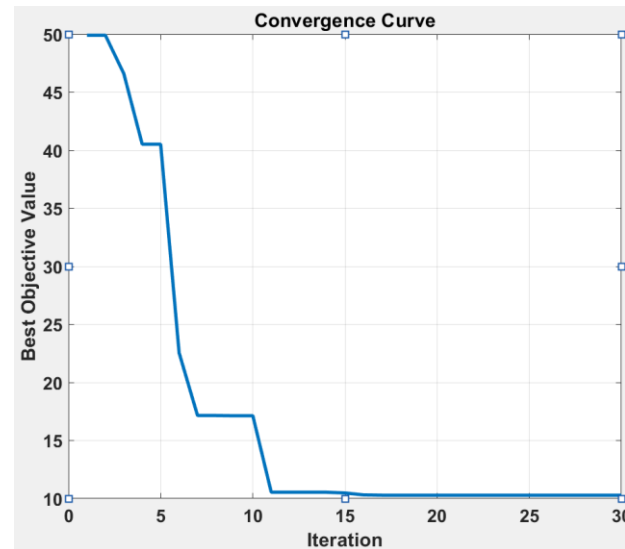
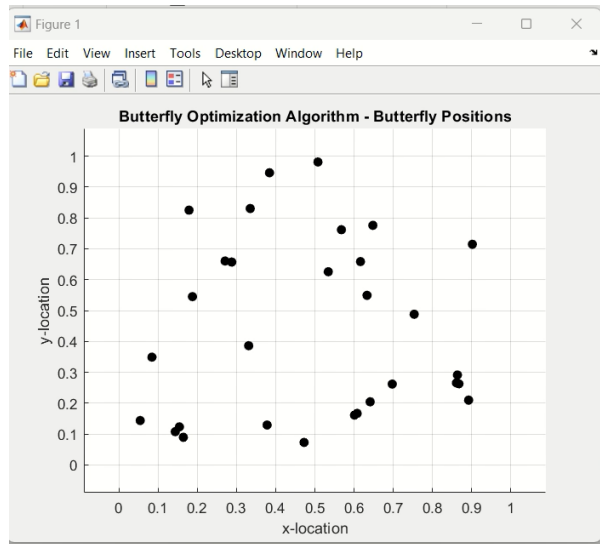


Objective function

$$F(x) = \sqrt{\frac{1}{n} \sum_{i=1}^n (f_i^* - f_i)^2}$$

Damage characterisation

Model	x	y	Damage length
40% mixed damage			
3D_FE	0.3	0.7	0.4
1D_3P	0.28	0.69	0.41
1D_5P	0.28	0.69	0.41
60% mixed damage			
3D_FE	0.2	0.8	0.6
1D_3P	0.19	0.79	0.62
1D_5P	0.19	0.79	0.59



60% mixed damage

40% mixed damage

Conclusion and further works

Conclusions

- Electric IPS with integrated (thermal and pressure) sensors provides benefits for real-time monitoring of aircraft parts to mitigate the effect of damage.
- Advanced IPS system is characterised by complex thermal-load induced deformation that may contribute to structural failure beyond the predictability of classical bending or shear behaviour.
- Analytical modelling framework offers excellent prospects for designing and SHM analysis of advanced ice protection systems.

Further works

- Comprehensive investigation on the prospects of electric IPS integrated with photonic sensors in line with the goals of PLEAIDES project.
- Advanced SHM analysis of electric IPS using sensor responses as objective parameters for damage detection, quantification and prognosis.

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- IMEC EUCLIA Belgium
- Politecnico di Milano Politecnico di Torino Italy
- Evictor SRO Canada
- Talos Analytics IKE ETS Greece

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